The Twentieth Annual Recreational Mathematics Conference at Lake Tahoe from “A” to “Z!”

By Larry Green, Lake Tahoe Community College

CMC³ will host the 20th annual Recreational Mathematics Conference on Friday and Saturday, April 22 and April 23 this year. The conference will be held in Lake Tahoe’s MontBleu Resort Casino and Spa, which is located near the lake and has all the amenities including a salon and spa, arcade, shopping area, and, of course, plenty of table games and slots if you are feeling lucky. This conference is unique in that all of the talks are recreational in nature, focusing on applications and other mysteries of mathematics.

The conference begins at 7:30 pm on Friday, April 22, with an opening get-together. Then Bruce Armbrust will take you across the galaxy as he shows you how mathematics is used to discover new exoplanets and figure out if little green men might be able to live there. On Saturday morning, the conference resumes with two sessions filled with more amazing uses, facts, and problems from mathematics. For example, you can see how math can be used to understand the art of self-defense or you can meditate with the spiritual side of mathematics. On Saturday after lunch, Paul Zorn will take elementary calculus to its extremes and show us some of the intricacies and depths behind integrals and derivatives. Two more sessions on recreational mathematics will follow Zorn’s talk where you will, for example, be able to learn all about the Chinese abacus and the eternal triangle. The grand finale of the conference will be this year’s student keynote presenter. If you have a student who may be interested in being this year’s Tahoe Student Speaker, please encourage them to apply. The student speaker will talk for 20 to 30 minutes. The committee will begin reviewing the applications

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Lake Tahoe Conference
(continued from front p. 1)
on March 1. Students can apply online at: www.cmc3.org/conference/callForStudentProposal.html.

On Friday evening, the CMC3 Foundation will be hosting its third annual conference gala, so be prepared for tasty morsels, amazing networking with other community college mathematics professors and a chance to help raise scholarship money for our students. Conference registration is $140 for members, $70 for adjunct instructor members, and $150 for nonmembers, ($80 for adjunct nonmembers). Registration will include a meal voucher of $15 toward any of the hotel’s eating establishments. This year we are offering a $70 rate for first-time attendees. Full-time students may register for the nominal fee of $5, which does not include the lunch voucher. For more information, contact your CMC3 campus representative or Larry Green, Tahoe Conference Program Chair, at DrLarryGreen@gmail.com. For the latest information and details about the conference and for the registration form please visit the CMC3 website at www.cmc3.org. With Armbrust and Zorn as our keynote speakers, you will all get to enjoy mathematics from “A” to “Z.”
President’s Report

Joe Conrad, CMC³ President, Solano Community College

I begin my term as your CMC³ president excited about what we have been doing and what lies ahead. CMC³ has been around for over forty years supplying quality professional development opportunities through our annual conferences. The Monterey conference continues to be one of the best conferences in the nation put on by a local mathematics organization. We are pleased with our transition to the Hyatt Regency Monterey Hotel. This has given us the ability to have all our sessions in one central area and hotel rooms with more benefits. We continue to attract top-notch speakers who inform our attendees of the latest trends in various areas of our profession. We look to continue this tradition in the future and are also hoping to implement online registration for the 2016 conference. As always, I encourage anyone reading this to put in a proposal to speak at the conference or, if you have a colleague or someone in mind who you would like to see on the program, please invite them to put in a proposal. The proposal form for the 2016 Monterey conference is already available at www.cmc3.org/conference/callForProposalsMonterey.html.

Both our conferences give great opportunities for community college mathematics faculty to learn about what’s happening in their profession and interact with colleagues from across the region. Of course, the fact that they are held in two of the most beautiful areas on the planet is not to be forgotten! Please come and join us this year in Tahoe and Monterey!

CMC³ History Quiz, Part 5

Mark Harbison, Sacramento City College

1. Rearrange the letters “Gristle” to spell the last name of the 2014 Monterey Keynote: James (Jim) _______.


3. In what year(s) did a Tahoe talk mention “knot” theory?

4. Chabot College hosted the first CMC³ conference in what month and year?

5. The 1974 Monterey keynote talk Design and Impact of Pocket Calculators was given by Dr. Thomas Whitney from which company?

(Answers are on page 5)
Math Nerd Musings: Power of the 18-Second Pause

Jay Lehmann, Newsletter Editor, College of San Mateo

“Any questions?” How many times have you asked your students this question? One hundred? One thousand? Ten thousand? In my twenty-five years of teaching, I’m sure I’ve surpassed the ten-thousand mark.

More importantly, in how many of those times did students respond with questions?

For my classes, not nearly enough.

There are outliers: that 1 class of students out of 20 that keeps firing questions throughout the hour. I can’t think of anything more cathartic. Sure, some questions would be at the low-level “What does that word say?” as my handwriting becomes tighter and tighter scribbles, but some questions would lead me to the very next thing I was going to discuss, and others would bring me to a dead stop because the questions would make me ponder something I’d never thought of before. How awesome!

I wish I could say for sure, but I can only make guesses. It could be that some students are busy copying the board for most of those seconds. It could be that some students are wrestling with whether to ask questions and it takes that long for their curiosity to win over their fear of embarrassment. It could be that some students to formulate good questions.

Collaborative learning’s a great way to address this issue, but it’s not always appropriate to use that methodology. Sometimes lecture-discussion is best.

This semester I’ve been blessed with having two student aides (supplemental instruction) for my prestatistics course. When they were trained, they were told to wait ten seconds after asking a question.

This caught my attention. I’d always heard three seconds. Or maybe it was five. But not ten.

So, I thought I’d give it a try. In my first attempt, I learned that ten seconds can be an eternity!

Weeks later, I learned that my discomfort was partly due to the fact that I’d actually been waiting even longer: 18 seconds. What was dumbfounding was that sometimes there would be complete silence for as long as 15 seconds, and then a student would raise their hand. I just couldn’t believe it could take that long. I mean, what’s going on during those 18 seconds, anyway?

Sometimes I invite questions with more flourish. After filling up a couple panels of the board, I’ll say, “Wow! We’ve covered a lot of ground here. And this stuff’s pretty hard and important. I bet you’ve got lots of questions.” I deliver this invitation sincerely and often a student asks a question within a couple of seconds. Usually, a second student will ask another question quite
quickly. No matter how many questions are asked, I’ll chill yet another 18 seconds to squeeze out all the questions I can.

So far, I’ve had mixed results. This experiment has been going marvelously in my first-semester calculus class, fairly well in my prestatistics class, and terribly in my trig class. In trig, they mostly stare me down for all that time. You can’t win ‘em all.

My worst fear with me encouraging questions is that students would ask lots of detailed, low-level questions that would slow the course down to a crawl. This fear was unfounded. Most of the additional questions students are posing are at a medium-to-high level.

In calculus, this is most evident. For example, students have asked me to illustrate why each hypothesis of a theorem is necessary. They’ve asked me to compare concepts from previous lectures with concepts in a current lecture. They’ve asked if the results of a theorem can be extended to other contexts. Students can never ask too many of these types of questions!

In the end, will my students learn more? It’s too early to say. But my calculus class has performed better on quizzes and the first test than any other calculus class I’ve had. Of course, this may be due to me inheriting an incredibly awesome group of students.

All I can say for sure is that I’m having one heck of a great time in calc. And I can’t help but think that my students are enjoying the course more, too.

Perhaps all of this is old news to you. If so, I applaud you for figuring out what took me 25 years to discover. Sometimes this column seems like a platform for me to confess my teaching failings and much-delayed epiphanies. But if your classes tend to be as quiet as mine were, you might want to give this a shot. But bring a timer and some resolve because your experience of 18 seconds may take on a completely different perspective.

### Answers to the CMC³ History Quiz, Part 5

(continued from page 3)

1. James (Jim) **Stigler** gave the 2014 Monterey keynote talk *Changing the Culture of Teaching: Mathematics Teaching and How to Improve It*. His research at UCLA includes the “Trends in International Mathematics and Science Study (TIMSS)”.

2. Ron Graham has so much talent both in mathematics and in **juggling** that he has been president of both the American Mathematical Society and the International Jugglers’ Association.

3. In 2014, Thomas Mattman (CSU Chico) gave the keynote talk *How to Tie a Knot (and Become Ruler of the World)*; In 2008, Jennifer Marie Mogel (UC Santa Cruz) gave the talk *Tying Knots and Folding Paper - a Fun Look at Some Unusual Areas of Math Research*; And in 1997, Theodore Stanford (U. Nevada, Reno) gave the talk *Knots, Links and Groups*.

4. In May, 1972, there were 63 people who attended the first (non-numbered) CMC³ conference. George Pedrick (CSU Hayward), Jim Hardesty (the NCTM Board of Directors), and Carol Kipps (Pasadena City College) were on the program.

5. Dr. Thomas Whitney from the **Hewlett-Packard Corporation** gave the 1974 CMC³ Monterey keynote talk.
Transform the Culture of Your Classroom

In Trigonometry: A Clever Study Guide author James Tanton provides a creative, readable, focused and connected overview of trigonometry. He expertly brings the Common Core Standards to life, provides a wealth of solved competition-level problems and supplies wonderfully open personal accounts of his own thinking through challenging tasks. It’s a fantastic resource for teachers of trigonometry.

—Dr. Ted Coe, Director, Mathematics, ACHIEVE, Inc.

Trigonometry: A Clever Study Guide
By James Tanton
Problem Book Series

This guide covers the story of trigonometry, providing a creative and coherent presentation of the subject based on understanding. Swift and complete, the guide is a fun read. The guide supplements and puts into perspective the material of any course on the subject one may have taken or is currently taking. It can even serve as a primary text for a first full course on the subject. The guide is a great resource for challenge problems (solutions provided!).

Educators at all levels of teaching can use this guide to help transform the culture of their classroom to one of natural self-reliant student thinking and problem solving.

Price: $19.95

To order, visit maa-store.hostedbywebstore.com or call 800-331-1622.
Grant. This was offered through the CMC³ Foundation courtesy of a donation from long-time member Wei-Jen Harrison. Because of this grant about 30 people, most of them first-timers, were able to attend the conference and stay in the hotel at a reduced rate.

We are already planning for next year’s conference, which will be held December 9–10, 2016. We will have both Friday and Saturday keynotes and our usual slate of sessions. Next year, your new president-elect, Katia Fuchs, will take the conference chair reins. I encourage all of you to consider presenting at the conference. Many of our best sessions come from our own members. Who else would have better wisdom for us? If you would like to present, please fill out a speaker proposal which can be found at the website: www.cmc3.org/conference/callForProposalsMonterey.html.

We hope to see you there and at the Tahoe conference in April!

Just as we are publishing this issue, we have received notification that one of our founding members, Sister Clarice Sparkman, has passed away at the age of 98. We plan to have a full article about her and her contributions to CMC³ in the summer newsletter.

Please consider putting one or two newsletters in the copy room for other instructors to read.
What’s Happening at the Reedley College STEM Math Study Center

Walid Tayar, Reedley College

The Reedley College STEM Math Study Center was born from an idea that students are more successful when they are part of a community that provides support and positive influence. This idea began to gain momentum in the fall and early spring of 2007/2008, and Reedley College math faculty began to seriously pursue the idea of creating a place on campus where a community could be created to help students become more successful in their math classes. They wanted to bring faculty offices together (formerly math faculty offices had been located in several buildings on campus) and also create a space devoted to math tutoring. The idea was well received on campus and was eventually written into a HSI STEM I grant that was awarded to the college in fall 2008.

The STEM MSC opened in spring 2009 in the RC library, its temporary location for two semesters. In its pilot semester (spring 2009), the center received 1319 visits by approximately 150 students from math, science, computer science, and engineering courses. Work began in the summer of 2009 on the conversion of a large group instruction classroom and wing of faculty offices. In spring 2010, with construction complete, the STEM MSC opened in its permanent location with Dr. Kathleen Landon as its coordinator.

Since opening in spring 2009, the STEM MSC has grown and changed in many ways. The most obvious change is the growth in the number of students served. In fall 2009, the STEM MSC served 296 students; as of this writing the STEM MSC has served 650 students in fall 2015. The tremendous growth of the STEM MSC can be attributed to the quality and effectiveness of its drop-in tutorial service. Students using the STEM MSC are 10-15% more likely to succeed in their math classes. In spring 2012, due to increased need for coordinator presence, Rebecca Reimer was hired as an additional part time coordinator. In fall 2012, the HSI STEM I grant ended and funding for the STEM MSC seamlessly transitioned to a second multi-million dollar HSI STEM II grant awarded to the college. Weekly basic skills workshops for Math 201 and Math 103 students were implemented in fall 2012 and spring 2013, respectively. In spring 2014, coordinators Landon and Reimer developed and implemented a CRLA compliant tutor training program to enhance the services provided by the STEM MSC.

The STEM MSC continues to grow and change as it establishes itself as a fundamental program at Reedley College. In spring 2016, RC will hire a full-time coordinator for the STEM MSC, and in fall 2016, the college district plans to pass a bond measure which will fund the relocation of the STEM MSC to a larger location where we can continue to serve RC students by creating a community of math and science learners supported and encouraged toward their full potential by an enthusiastic faculty. We look with great anticipation toward the future.
What’s Happening at San Jose City College
Kevin McCandless

It is an exciting time to be a member of the San Jose City College (SJCC) Mathematics Department! We provide excellent student support services and have a forward-thinking mathematics faculty that is dedicated to helping our students reach their goals. We have also been granted TWO new tenure-track positions! So, in the words of the great Bob Dylan, “the times they are a-changin!”

Our Metas Program is thriving and just secured another Title V: Hispanic-Serving Institutions grant from the U.S. Department of Education. Metas, which is Spanish for goals, is a program that provides its participants with student support services, such as Supplemental Instruction, Peer-Led Team Learning, textbook & calculator loans, and counseling services. Under the leadership of director Robert Gutierrez, the Metas Program has grown to include a Summer Bridge and First-Year Experience learning community that facilitates the transition of local high school students to SJCC, many of whom are the first in their family to attend college.

Additionally, under the fearless leadership of our second-year Dean of Mathematics & Science, Jamie Alonzo, we have embarked upon a meaningful strategic planning process aimed at increasing student learning and success. This semester, we held our first-ever mathematics retreat at a local winery, where small teams presented their findings from literature reviews in key areas of math education, including contextualized instruction and alternate pathways through developmental mathematics. In addition to great wine, we also experienced great conversations among colleagues and emerged with the following priorities:

- Offer a two-semester alternative for beginning and intermediate algebra (part A / part B)
- Offer contextualized mathematics courses that meet the needs of our technical education students
- Join Statway from the Carnegie Foundation for the Advancement of Teaching
- Begin implementing the STEM Core Network from Growth Sector, which includes an accelerated pathway from beginning algebra to calculus
- Revise the procedure by which students are placed into developmental mathematics courses
- Develop a culture of teamwork among full-time and part-time math faculty in which syllabi, course calendars, and assessments are commonly shared
- Review our process of SLO (student learning outcomes) assessment and discuss how we define “success” for our students

Since our retreat, we have divided our department into small working groups to tackle each item on the list; every faculty member is

(see “San Jose City College” on p. 11)
Before Apollonius of Perga gave us the names for the conic sections that we use now, Archimedes and his predecessors generally referred to the conics as a section of a right-angled cone (parabola), a section of an obtuse-angled cone (hyperbola), and a section of an acute-angled cone (ellipse) [3, p. 113]. This terminology can be found in Book XI of Euclid’s Elements [1, p. 368], where he defines a cone as follows.

18. When, one side of those about the right angle in a right-angled triangle remaining fixed, the triangle is carried round and restored again to the same position from which it began to be moved, the figure so comprehended is a cone. And, if the straight line which remains fixed be equal to the remaining side about the right angle which is carried round, the cone will be right-angled; if less, obtuse-angled; and if greater, acute-angled.

According to Euclid, then, to obtain a cone, we revolve a right triangle about one of its legs. See Figure 1. Now, if the fixed side (leg) equals in length the remaining side (leg), then $\angle \alpha = 45^\circ$, so that the apex angle of the cone is a right angle (a right-angled cone); if the fixed side is less than the remaining side, then $\angle \alpha > 45^\circ$, so that the apex angle of the cone is obtuse (an obtuse-angled cone); and if the fixed side is greater than the remaining side, then $\angle \alpha < 45^\circ$, so that the apex angle of the cone is acute (an acute-angled cone). Cutting a cone by passing a plane through the cone orthogonally to its lateral side, then, produces a section of the cone along the trace. See Figure 2.

Apollonius’s innovative approach to conic sections was first to remove two restrictions. Apollonius allowed himself to use any cone, and not necessarily a right circular cone—in fact, double-napped cones—and he allowed himself to cut a double-napped cone by a plane at any angle to a lateral side. See Figure 3. In this way, Apollonius obtained the different conic sections from a single (double-napped) cone instead of three different cones by varying the angle of the cutting plane as we do today.

Now, the terms “parabola,” “hyperbola,” and “ellipse” come from the Pythagorean problem of the application of areas. (See [2, pp. 150–154], for example.) We hint here why Apollonius used these terms to name the conic sections.

In the following figure, a certain area equals or is applied to (Greek: parabola) another certain area.
In the following figure, a certain area exceeds (Greek: hyperbola) another certain area.

And, in the following figure, a certain area falls short of (Greek: ellipse) another certain area.

(To be continued.)

Previous columns are on the Web at <http://ms.yccd.edu/history-glass.aspx>.

References


AMATYC’s Steve Blasberg Faculty Math League Award
Barbara Illowsky, De Anza College

Most of you know Steve Blasberg as one of CMC3’s long-serving At-Large Board Members. Steve’s served in that position since the mid-1990’s, doing a variety of tasks. In fact, even though Steve retired in 2013 after 39 years of teaching math, 38 of them at West Valley College, he is still on our Board. What you might not know is that Steve continues to remain active in the profession by writing and/or compiling several math competitions each year, including two local competitions sponsored by the Santa Clara Valley Math Association, as well as the Student Math League competition sponsored by AMATYC. Steve was the Test Developer for AMATYC’s competition from 2000 – 2015. In fact, he would continue to be the Test Developer except that he has termed out.

At each annual AMATYC conference, there is another competition. Each year, a few dozen faculty spend an afternoon tackling their own set of mathematics problems. Unlike the student competition, the faculty one does not lead to scholarships; the participants do it “for fun.” At the November 2015 AMATYC national conference, AMATYC announced that it was renaming the top prize in this annual competition to be the “Steve Blasberg Faculty Math League Award.” The award naming is a recognition that Steve has put more time and effort into the exam than anyone else and for many years.

Congratulations, Steve!!!

The Pleasures of Problems
Kevin Olwell, San Joaquin Delta Community College

Spring 2016 Problem: The local youth soccer league is planning its annual tournament. The tournament director has asked you to pair the teams for the first round of play. There are 20 teams. How many ways can the 20 teams be paired?

Fall 2015 Problem: Find the value of the infinite product $P$ whose factors are

$$(n^3 - 1)/(n^3 + 1) \text{ for } n = 2, 3, 4, \ldots$$

Solutions to the Fall Problem were submitted by Fred Teti, Paul Cripe, Carlos Valencia and Joe Conrad.

Let $P_n$ denote the product of the factors from 2 through $n$. The solutions simplified $P_n$ by factoring the numerator and denominator. Many of the factors in the numerator and denominator now cancel each other. The result is

$$P_n = \frac{2}{3} \frac{n^2+n+1}{n^2+n} \Rightarrow P = \lim_{n \to \infty} P_n = \frac{2}{3}.$$  

All are invited to submit a solution to the Spring 2016 problem either via email or US mail at the address below.

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CMC3 Foundation Report
Debbie Van Sickle, Foundation Past-President, Sacramento City College

New CMC3 Foundation Board of Directors
At the end of 2015, I completed eight years on the CMC3 board of directors, including four years as president. It has been a great pleasure to work with many wonderful people with the end result of awarding scholarships to talented community college students to help them with the next stages of their education. I would like to thank outgoing board members Paine Ngai, Karl Ting and Danny Tran for all their help and support. I would especially like to thank our outgoing treasurer, Rebecca Fouquette, who served with me for six years. I could not have done the job without her. I would like to end my service by introducing you to the new board of directors:

Foundation President: Mark Harbison, Sacramento City College

Foundation Treasurer: Leslie Banta, Mendocino College

Foundation Member-at-Large: Shawn Lanier, Woodland College

Foundation Member-at-Large: James Sullivan, Sierra College

Scholarships and Competitions
Last spring, the CMC3 Foundation awarded a total of $6,000 in scholarships to students attending four of our member colleges. Last year’s winners can be seen on the foundation scholarship web page www.cmc3.org/foundation.html#scholarships

We are now beginning our 2016 scholarship competition. In May, we will be awarding one $3000 first place, one $2000 second place, and one $1000 third place scholarship to highly qualified community college students. Our winners will be students who have successfully completed a minimum of 30 college units, including at least 8 units at CMC3 colleges, are currently enrolled in a minimum of 6 units at a CMC3 college, and who have completed at least one mathematics course at the level of second semester engineering calculus or higher. Application packets will be due in March 16, 2016. Instructions and application materials will be sent to campus reps soon or you can go to www.cmc3.org/foundation.html#scholarships.

During the Monterey conference, the Foundation sponsored a student poster contest that included a $75 cash prize and lunch with conference attendees. This year’s winner was Kevin Perez from Solano Community College. His poster was titled “Justification of Second Derivative Tests from Optimization and Extension to More than Two Variables”.

The Spring Conference at Lake Tahoe will feature a talk given by the winner of the Foundation’s annual Student Speaker Competition. Thanks to the generous sponsorship of Debra Landre, a former CMC3 President, the winner of this competition will be given a $500 scholarship.

Applications for both competitions are open to any currently enrolled community college student in our region. More information about these competitions will be available on our website at www.cmc3.org/foundation.html.
Fundraising

CMC³ Foundation scholarships are only made possible because of the generosity of our members, our vendors and other contributors. At the Monterey conference this year, we raised almost $4,000 in a combination cash donations, and the sale of raffle tickets and merchandise.

At the spring conference in Lake Tahoe, we will be having our third annual Gala fundraiser on Friday, April 22, at 9:00 p.m. after the keynote speaker. Tickets for a suggested donation of $20 will be available at the registration table and at the door. We will serve alcoholic and non-alcoholic beverages, and a variety of hearty Hors d'oeuvres and deserts. Your ticket will also be put in a drawing at the end of the conference for prizes including free registration to next year’s conference in Monterey and a free stay at the MontBleu Hotel.

Everyone who is a member of CMC³ is also a member of the Foundation. Our members and other supporters can help us continue our scholarship programs by supporting us in the following ways.

Make a tax-deductible cash contribution*. Donate prizes for our raffle. The value of these items is also tax-deductible*. Donations can include (but are not limited to):

- Wine, beer, and other libations
- Candy, cookies and other non-perishable food items
- Gift cards for stores, restaurants, or services
- New items you received as a gift and can “re-gift” to us (stationary, books, t-shirts, electronics etc.)
- New gift baskets (store-bought or homemade)
- New items we can add to other gift baskets
- Baskets (need not be new) we can use to make gift baskets
- Help us get cash or raffle prize donations from businesses or individuals. I especially would like help reaching out to publishers and other vendors that you may have an especially good relationship with.
- Purchase lots of tickets for our raffle and encourage your friends to do so as well.
- Purchase our t-shirts and other items for sale at our table during the conferences.
- Attend our Gala fundraiser on Friday night of the Tahoe conference.
- Volunteer to help us with any of our fundraising efforts (sell raffle tickets or gala tickets, help with event set up or clean up, put prize baskets together, etc.)
- Suggest new fundraising ideas to any member of our board.

I would like to thank everyone who made generous donations of money and prizes over the last year. Without your support none of our work would have been possible.

* CMC³ Foundation is a nonprofit charitable organization under section 501(c)3 of the Internal Revenue Code. Contributions are tax deductible to the extent allowable under federal law (as long as no goods or services are provided in exchange for the donation). Our Tax Identification Number is 94-3227552. Cash donations can be made in three ways:

- At the time you register for either conference (There is a box to check on the registration form. Please use a separate check, but mail it in the same envelope as your registration form.)
- In person at one of our conferences, either by check, cash, or credit card.
- By mailing a check to our treasure, Leslie Banta at Mendocino Community College, 1000 Hensley Creek Rd, Ukiah, CA 95482
Bull: Peer Assessment Assessed

Seattle, Thursday, August 13, 2015: JSM

On the last day of the Joint Statistical Meetings, in very nearly the last session, there was a paper presented on peer assessment in statistics teaching that has important implications for community college mathematics and statistics instructors. (Lesson: do not ignore the last sessions of conferences!) The author of the paper was Dennis Sun, recently hired at Cal Poly, but at that time at Google, having completed his doctorate in statistics at Stanford. The paper reported a project evaluating the use of peer assessment in teaching introductory statistics classes at Stanford.

My purpose here is to report on that project and discuss how peer assessment can be used in community college mathematics and statistics teaching. If peer assessment is a good idea, what are the challenges in using peer assessment in community college teaching? Can the model used at Stanford be used in community colleges? The answer is “under certain conditions” and sparingly. But to see what conditions are and why the word “sparingly” is used, we will first see how peer assessment was used in the very well thought project at Stanford. Those who have used peer assessment in community college teaching may wish to compare their experiences with what was done at Stanford; it may be that a different model altogether is needed.

Peer Assessment defined and refined

Peer assessment is having students grade one another’s work. But that should raise the questions: What work? Any and all work? The team at Stanford, in implementing peer assessment, thought the matter through and recognized that peer assessment is best used “restrictively”, and not for any or all kinds of assignments. The figure here comes from Sun’s power point; his perspective is that tests and projects are too high stakes for students and too complicated to mark for peer assessment. On the other hand (and not in the power point shown), many mathematics and statistics exercises that involve either calculation or possibly multiple-choice responses can be reliably and efficiently graded by software. And these too are not good candidates for peer assessment; it would be time wasted. However, there are exercises that require short, free-response answers, and these are complex enough so that they cannot be graded by software. Moreover, these questions requiring more complex answers are precisely the kinds of questions in statistics teaching that we can argue should be a part of every student’s experience. At the very least, students should be required to freely compose an answer rather than only choosing from a list of provided choices. For statistics, often (though not exclusively) the questions involve interpretation in the context of a statistical question. But they can also be questions whose prompt start with the word: “explain.” Responses to questions that require human graders are also found in mathematics teaching as well as statistics education; think of the kinds of things one looks for in the
solution to an optimization exercise. In all of these examples, the responses are not inherently hard to grade but the grading can be time consuming, and the grading typically cannot be done using software. Moreover, the complexity of the responses required are the sort of questions to be asked of students that we do not wish to sacrifice on the altar of expediency or efficiency of grading. They are too valuable for students. Sun’s insight, (which we are taking up here) is that questions involving moderately complex responses are excellent candidates for peer assessment; if students doing the assessment are given a grading rubric as well as sample answers, those students learn what their instructors think is important for a response, and learning what instructors think is important is valued.

Sun’s JSM presentation focused on using peer assessment in the classroom, but the project at Stanford actually involved a carefully designed randomized study that compared the outcomes of for topics covered using peer assessment with topics where peer assessment was not used. The study showed that peer assessment did increase test scores not only significantly, but the authors argue, substantively (See Sun et. al. 2015).

Peer assessment, if it can be done successfully would thus appear to have a lot going for it. Using peer assessment provides students with valued and important instruction (“what really is important here”) for the answers to free response questions that are more complex than single answers or numerical answers. Moreover, these are the kinds of questions that we, as mathematics and statistics educators, think are important, but which are tedious to grade. We want students to have the experience of grappling with expressing ideas and concepts in a complete way, and peer assessment offers the promise of their efforts being examined by humans. There is evidence from the randomized study that peer assessment has a positive effect, at least at Stanford. So, what needs to be done to successfully implement peer assessment? And will what is needed work in community colleges? We have come up with what we think are three necessary elements for workable peer assessment.

**Specifications for workable peer assessment: online implementation and rubrics**

For a workable peer assessment system, one appears to need: (a) an online learning management system (an LMS), (b) rubrics developed for each of the questions to be graded by students, and (c) sufficient student incentives. The first two of these are discussed in this section, followed by two examples, and that followed by a discussion of incentives.

In the Stanford study each student graded and gave feedback to three other students’ responses to questions, and in turn each student was graded and received feedback from three other students. Moreover, each student gave (and received) evaluations of the feedback received as to its usefulness. The actual grade each student received was the median of the three grades, to prevent arbitrary sabotaging by peer assessors. All of the “exchanges” of questions were anonymous to comply with FERPA privacy regulations for student grading, so that no student was able to know whose response he or she was grading, or which students had graded his or her response, and all of this was facilitated by an online LMS. (In the Stanford study, instructors also graded the responses, but this was for the purpose of the randomized study.) Effectively exchanging responses to be marked for peer assessment, and doing so anonymously (which is necessary by law!) would appear to be only feasible using an LMS.
Using an online LMS may or may not be a significant challenge for the implementation peer assessment in community colleges. In many community colleges, each instructor makes all the decisions for his or her class of between 25 and 45 students. This means that each instructor may be responsible for figuring out how the online LMS needs to be configured to exchange responses, unless that instructor can depend on help from colleagues. If the instructor is using a publisher’s online system, that system may or may not encourage open-ended responses. Things may well be easier where a team of instructors share responsibilities over several sections; it is likely that one of the team will be able to take on the task of making an LMS do the anonymous exchanges of responses necessary, and keeping track of the grades. There are online helps for implementation in Moodle and Canvas. Working as a team with the same syllabus also means that the pool in which exchanges happen may be larger; students can be grading students from other sections in that scenario, if the LMS being used allows that.

Similar comments about teamwork apply to making rubrics: it is more efficient if the labor of making marking rubrics can be spread out amongst members of a team rather than having to be done by each instructor individually. (The same comment can apply to making tests and quizzes, where there again are many advantages to working in a team, and the end product is often better than the sum of individual efforts; that teamwork is not valued enough in our working culture is a lament that has been voiced in these Bull columns previously.) We give two examples of rubrics.

**Example for Statistics: Treatment for Cocaine Addiction**

This example is from an exercise found in Lock et. al. (2013: 485). The exercise uses data from an experiment in which 72 cocaine addicts seeking treatment were randomly assigned to one of three treatment groups for six weeks. In the exercise, the numbers of addicts who had suffered a relapse into addiction in the six weeks were given. Students were asked to create a two-way table, and if appropriate, carry out a chi-square test. It appears that this could be done by hand with the aid of a calculator or possibly using software, but if using software, the results could be as shown (here, using Fathom).

One part of the exercise asks students to “give an informative conclusion” to the test. This is just the sort of question (we have been arguing) for which peer assessment is appropriate. When students are to begin assessing their peers, they should be given both a suggested complete answer and also a rubric to guide the marking.

Often students answer “an informative conclusion” question with too little information and ambiguous references, such as: “it is statistically significant” begging the
question of what is statistically significant, and more importantly, not connecting the
calculated output to the question asked. Having the rubric alerts the student to what the
standards are for these kinds of questions. I have specified four points for this example,
but it could be eight or ten points. One could specify penalties for making common
errors, although I think that a positive approach is probably better. Notice that a question
about whether the Chi-square test is appropriate could be treated in a similar fashion,
with students required to give the reason for the answer.

**Sample Answer**

The low $p$ value of 0.0052 shows that the hypothesis test is statistically significant,
and we have evidence against the null hypothesis that relapses are independent of
the type of drug used. We have evidence that there is some association between
the type of drug used and the probability of a relapse into addiction in a six-week
treatment period.

**Rubric for Grading**

This question is worth four points, according to the guidelines given here:

1. point for stating the correct conclusion to the test.
2. point for giving a reason for coming to the conclusion.
3. points for stating the conclusion in the context of statistical question posed.
   Mention should be made of drugs being used to combat cocaine addiction, and
   specifically preventing relapses.

**Example for Mathematics: An optimization problem**

The second example is from Stewart’s *Calculus* (2008:329). The exercise reads:
A cylindrical can without a top is made to contain $V$ cm$^3$ of liquid. Find the dimensions
of the can that will minimize the cost of the metal to make the can.

**Sample Answer**

Minimizing the cost means minimizing the surface area $S=2\pi rh+\pi r^2$ of the can
subject to a constraint of a given volume $V$, where $r$ is the radius of the can, and $h$
is the height of the can.

\[ V = \pi r^2 h \] so \[ h = \frac{V}{\pi r^2} \] and hence

\[ S(r) = 2\pi r \left( \frac{V}{\pi r^2} \right) + \pi r^2 \]

\[ = \frac{2V}{r} + \pi r^2 \]

Thus, \( S'(r) = \frac{-2V}{r^2} + 2\pi r \) and

\[ S'(r) = \frac{-2V}{r^2} + 2\pi r = 0 \] when

\[ r^3 = \frac{V}{\pi} \] and so \[ r = \sqrt[3]{\frac{V}{\pi}} = \left( \frac{V}{\pi} \right)^{\frac{1}{3}} \]

Now, \[ h = \frac{V}{\pi \left( \frac{V}{\pi} \right)^{\frac{1}{3}}} = \left( \frac{V}{\pi} \right)^{\frac{1}{3}} \]
So, the critical value is \[ r = h = \left( \frac{V}{\pi} \right)^{\frac{1}{3}} \]
Since \( S'(r) = \frac{4V}{r} + 2\pi > 0 \) for all \( r > 0 \) the function \( S = 2\pi rh + \pi r^2 \) is concave up for all \( r > 0 \), so the critical value must be a minimum, and the minimum amount of material is where \( r = h = \left( \frac{V}{\pi} \right)^{1/3} \).

**Rubric for Grading**

This question is worth eight points, according to the guidelines here:

1 point for appropriate labeled picture
1 point for defining the situation and the variables used
1 point for correct optimization and constraint equations
1 point for correct substitution making the optimization equation a single value equation.
1 point for correct differentiation of optimization equation
1 point for critical value found
1 point for determining and showing that the critical value is a minimum
1 point for stating the answer in terms of the question posed

The reader may well have written the sample answer or the rubric differently. Fair enough; but the point is that guidance is given as to what kinds of answers are expected; in other words, what is important. It is probably also important to give students option to appeal the peer grading if the student thinks the grading has not been accurate. That leads us to the third specification.

The third specification – sufficient student incentives – is perhaps the most important for community college implementation.

**Specifications for workable peer assessment: sufficient student incentives**

In the Stanford study, assessing peers was made to count ten percent of the total grade of the course. That kind of carrot/stick is probably necessary to make any kind of peer assessment work; whether it should be ten percent or some other number probably depends on the “grading structure” that is being used, but the proportion has to be big enough so that ignoring the peer assessment is not a good idea for students. That each student is assessed by three students makes it more unlikely that any student will not get a grade at all. But making peer assessment part of the grade, while necessary, is probably not sufficient incentive scaffolding for the project to work for community college students. That peer assessment involves grades will raise student anxiety automatically for some students, “grade” being a hallowed term. The policies
regarding how much the participation in peer assessment will “count” and also how much the peer assessed material will count are matters that need to be carefully thought through in the context of the overall grading policies being used. On the one hand, one wants both to count enough that participation will be very high, and not count so much to itself increase grade anxiety greatly, especially (as we are emphasizing) the assessment is to be useful as an instructional tool. Probably, what is assessed by peers should constitute not a large proportion of all the assessment that is done. A good balance here is important.

An immediate concern for students is that the grading will be unfair. It may be that instructors have similar fears about fairness; as a point of interest, the Stanford study generally found that peers were more generous than the instructors (See the graphic above). The policy of using the median grade of three graders also appears to be a good idea. Students may also think that peer assessment will be a waste of time (“Shouldn’t teachers be doing the grading? That’s their job!”). Peer assessment has to be introduced and used in a way that fears of unfairness and time wasting will not be a problem.

For these reasons, we suggest that peer assessment should be focused on the kind of open-ended questions whose responses are seen as problematic by students themselves, where what a good answer is may not be completely clear. And with this, what can be emphasized is that the experience of grading according to a rubric will be a good way of learning about what is expected in the kinds of questions that are not machine graded. Helping students view peer assessment as something akin to working “practice tests” that give a partial experience of what they will face when actually being assessed may be helpful. The Stanford study also required students to give some kind of written feedback as well as a numerical score to the students whose responses they were grading. This is a valuable part of the exercise.

Another aspect hard to judge in advance, but still important is the amount of time the assessment will take most students. Again, it should not be onerous, but at the same time, not trivial.

Conclusion

One of the first reactions to Sun’s JSM paper when discussing it with community college instructor colleagues has been: “Well, it works at Stanford, but I don’t think it will work with our students.” One thing these colleagues may have in mind is that some proportion of community college students are still in the process of learning to be students, and learning to do so in the context of many other commitments. Extra innovations such as peer assessment may be seen by many students as unwanted, certainly unbidden, and unnecessary additions to attaining their goals efficiently and easily. And yet, part of learning to be a student is to recognize when doing something that certainly will take a bit of effort will yield benefits down the road, or that there are activities besides memorizing and doing problems that can be helpful. It is with these concerns in mind that the comments above have emphasized using peer assessment as a teaching tool that at the same time solves the problem of giving feedback to students. That peer assessment relieves the instructor’s marking burden has not been emphasized here; indeed, we would venture to say that reducing the marking burden should not be the sole reason for using per assessment. Using peer assessment effectively requires a good deal of planning and thought for its implementation; Sun’s insight that peer assessment makes most sense for questions with responses of “intermediate” complexity is just the
first step in thinking things through. If peer assessment can be used to give feedback to students where otherwise it would be not possible to give feedback, and if peer assessment can also be used as an instructional tool, then there is good reason put in the effort to use it.

Ken Bull


Calendar

February 26–27, 2016: Joint Meetings of the MAA-Florida Section and FTYCMA, Saint Leo University, Saint Leo, FL. Contact: Altay Ozgener. Website: http://sections.maa.org/florida/newsletter/callslu.htm

February 27, 2016: MAA Golden Section Meeting, UC Davis. Contact: Chris Goff cgoff@pacific.edu. Website: http://sections.maa.org/golden/

March 4–5, 2016: CMC³-South 31st Annual Conference, Kellogg West Conference Center & Hotel near Cal Poly Pomona. Contact: Maribel Lopez (310) 434-4000 x-3484, lopez.maribel@gmail.com

March 10-13, 2016: 28th Annual International Conference on Technology in Collegiate Mathematics (ICTCM), Atlanta, GA. Contact: Joanne Foster (800) 472-6288 or (207) 676-8688, email: joanne.foster@pearson.com

March 11–12, 2016, CMC Central Conference, Bakersfield CA. Website: http://cmc-math.org/about-2/

April 1, 2016. INMATYC Spring Conference. Indiana University/Purdue University at Indianapolis. Contact: Becky Pohle. Website: http://irmc.matyc.org/

April 8–10, 2016: 2016 NYSMATYC Annual Conference, Kingston, NY. Contact: Josh Hammond. Website: www.nysmatyc.org

April 13-16, 2016 NCTM 94th Annual Meeting, San Francisco, CA. Contact: NCTM Office (703) 620-9840, email: annlmtg@nctm.org

April 22-23, 2016: 20th Annual Recreational Mathematics Conference, MontBleue Hotel. Contact Larry Green (530) 541-4660 ext. 341, drlarrygreen@gmail.com

July 24-31, 2016 13th International Congress on Mathematical Education (ICME-13), Hamburg, Germany. Contact: Gabriele Kaiser, +49 40 42838 5320 x-5321, email: contact@icme13.org

October 26-28, 2016 NCTM Western Regional Meeting, Phoenix, AZ. Contact: NCTM Office (703) 620-9840, email: regconf@nctm.org

November 4—5, CMC South Conference, Palm Springs Convention Center. Website: http://www.cmc-south.org/

November 17—20, AMATYC 42nd Conference, Denver, CO. Contact: Keven Doctor, e-mail: keven.dockter@anokaramsey.edu


December 9–10, 2016, CMC³ 44th Annual Conference, Hyatt Regency Monterey Hotel and Spa, Monterey, CA. Contact Katia Fuchs, City College of San Francisco, (510) 325-1616, efuchs@ccsf.edu

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